Primer Examen Parcial

Temario **A**

Prof. *José Saquimux* Aux. *José Márquez*

Tema 1 Operaciones con complejos

$$z_1 = \frac{2i^{28} - i^{19}}{3i - 1}, \quad z_2 = \frac{1}{2} - \frac{\sqrt{3}}{2}i, \quad z_3 = 1 - 2i$$

a. $\mathrm{Re}[z_1]$

$$z_1 = \frac{2i^{28} - i^{19}}{3i - 1} = \frac{2(i^2)^{14} - i(i^2)^9}{3i - 1} = \frac{2(-1)^{14} - i(-1)^9}{3i - 1} = \frac{2 + i}{3i - 1}$$

$$z_1 = \frac{2 + i}{3i - 1} * \frac{-3i - 1}{-3i - 1} = \frac{-6i - 2 - 3i^2 - i}{9 + 1} = \frac{-2 + 3 - 7i}{10} = \frac{1 - 7i}{10} = \frac{1}{10} - \frac{7}{10}i$$

$$\boldsymbol{Re}[\boldsymbol{z_1}] = \frac{\mathbf{1}}{\mathbf{10}}$$

b. $i{z_2}^2 - z_3$

$$i{z_2}^2 - z_3 = i\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)^2 - (1 - 2i) = i\left(1e^{-i\frac{\pi}{3}}\right)^2 - (1 - 2i) = \left(e^{i\frac{\pi}{2}}\right)\left(e^{-i\frac{2\pi}{3}}\right) - (1 - 2i)$$

$$i{z_2}^2 - z_3 = \left(e^{i\frac{\pi}{2} - i\frac{2\pi}{3}}\right) - (1 - 2i) = \left(e^{-i\frac{\pi}{6}}\right) - (1 - 2i) = \left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right) - (1 - 2i)$$

$$i{z_2}^2 - z_3 = \frac{\sqrt{3}}{2} - \frac{1}{2}i - 1 + 2i = \frac{\sqrt{3} - 2}{2} + \frac{3}{2}i$$

$$\boldsymbol{i{z_2}^2 - z_3} = \frac{\sqrt{\mathbf{3}} - \mathbf{2}}{\mathbf{2}} + \frac{\mathbf{3}}{\mathbf{2}}\boldsymbol{i}$$

c. Valor principal en cartesianas $(1 + j)^5(1 - j)^{1/2}$

$$(1 + j)^5(1 - j)^{1/2} = \left(\sqrt{2}e^{j\frac{\pi}{4}}\right)^5 (\sqrt{2}e^{-j\frac{\pi}{4}})^{1/2} = \left(\sqrt{2}\right)^5 e^{j\frac{5\pi}{4}} (\sqrt{2})^{1/2} e^{j\left(\frac{-\frac{\pi}{4} + 2\pi k}{2}\right)}$$

$$para\ k = \{0, 1\}, \qquad tomando\ k = 0$$

$$(1 + j)^5(1 - j)^{1/2} = \left(\sqrt{2}\right)^{11/2} e^{j\frac{5\pi}{4}} e^{-j\frac{\pi}{8}} = \left(\sqrt{2}\right)^{11/2} e^{j\left(\frac{5\pi}{4} - \frac{\pi}{8}\right)} = \left(\sqrt{2}\right)^{11/2} e^{j\frac{9\pi}{8}}$$

$$(1 + j)^5(1 - j)^{1/2} = \left(\sqrt{2}\right)^{11/2}\left[\cos\left(\frac{9\pi}{8}\right) + j\sin\left(\frac{9\pi}{8}\right)\right]$$

$$(\mathbf{1} + \boldsymbol{j})^{\mathbf{5}}(\mathbf{1} - \boldsymbol{j})^{\mathbf{1/2}} = \left(\sqrt{\mathbf{2}}\right)^{\mathbf{11/2}} \mathbf{cos}\left(\frac{\mathbf{9}\boldsymbol{\pi}}{\mathbf{8}}\right) + \boldsymbol{j}\left(\sqrt{\mathbf{2}}\right)^{\mathbf{11/2}} \mathbf{sin}\left(\frac{\mathbf{9}\boldsymbol{\pi}}{\mathbf{8}}\right)$$

$$(\mathbf{1} + \boldsymbol{j})^{\mathbf{5}}(\mathbf{1} - \boldsymbol{j})^{\mathbf{1/2}} = -\mathbf{6.215} - \boldsymbol{j}\mathbf{2.574}$$

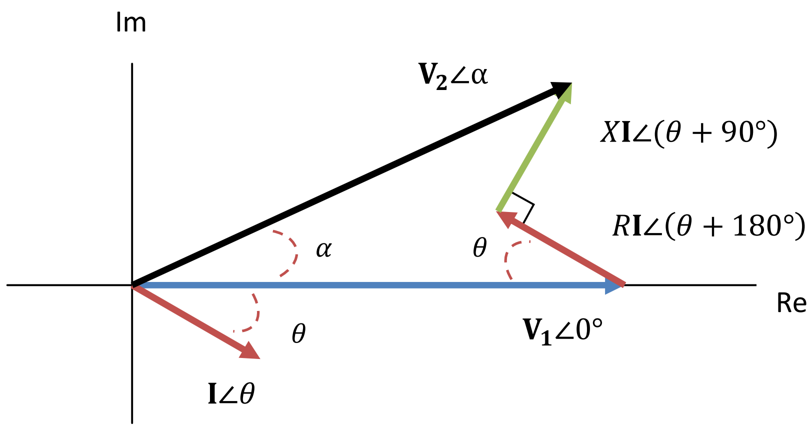
Tema 2 Lugar geométrico y fasores

a. Dibuje el diagrama fasorial de la ecuación compleja,

$$\mathbf{V_2} = \mathbf{V_1}\angle 0° - (R - jX)\mathbf{I}\angle\theta \quad suponga\ \theta < 0$$

$$\mathbf{V_2} = \mathbf{V_1}\angle 0° - R\mathbf{I}\angle\theta + jX\mathbf{I}\angle\theta$$

$$\mathbf{V_2} = \mathbf{V_1}\angle 0° + R\mathbf{I}\angle(\theta + 180°) + X\mathbf{I}\angle(\theta + 90°)$$



b. Mostrando los vectores que la generan, dibuje el lugar geométrico de

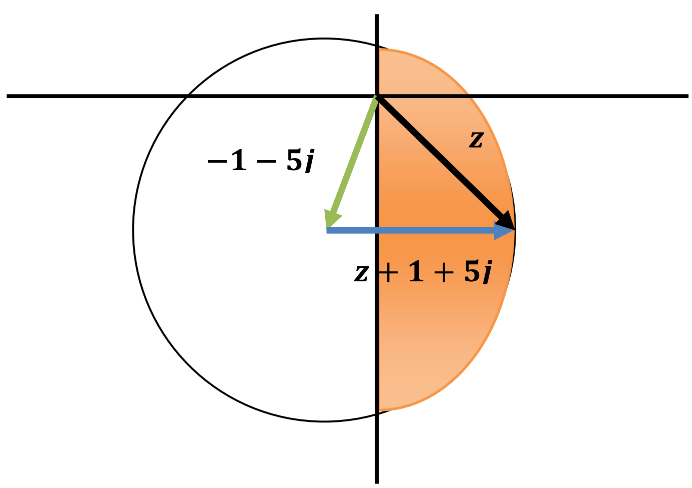
$$|z + 1 + 5j| \leq 7 \quad si\ \mathrm{Re}[z] \geq 0$$

$$z = x + jy$$

$$|x + jy + 1 + 5j| \leq 7$$
$$|(x + 1) + j(y + 5)| \leq 7$$
$$|(x + 1) + j(y + 5)|^2 \leq 7^2$$

$$(x + 1)^2 + (y + 5)^2 \leq 7^2$$
$$Circulo\ de\ radio\ 7$$
$$con\ centro\ en\ (-1, -5)$$

$$\boldsymbol{|z + 1 + 5j| \leq 7 \quad si\ \mathrm{Re}[z] \geq 0}$$
$$\boldsymbol{El\ lugar\ geométrico\ es\ la\ region}$$
$$\boldsymbol{sombreada\ dentro\ del\ círculo.}$$

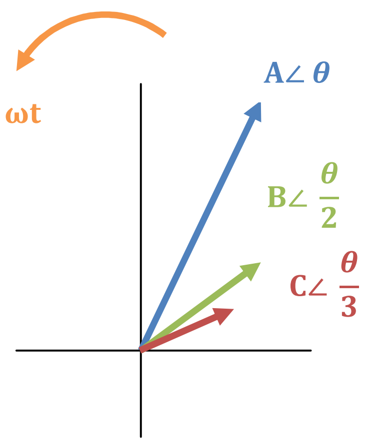


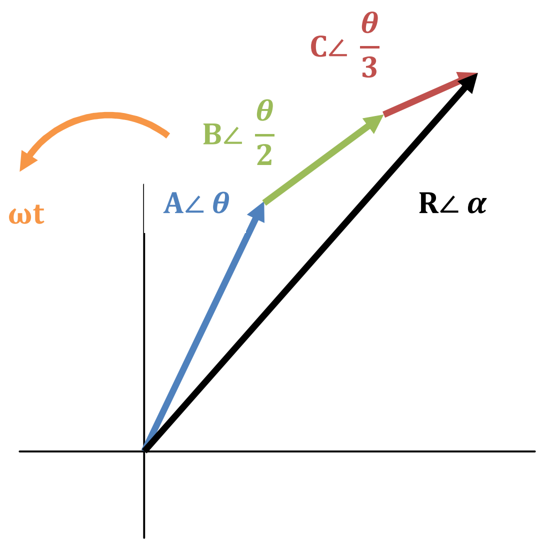
c. Determine los fasores y dibuje la suma fasorial asociada a la suma trigonométrica,

$$10\sin(\omega t+\theta)+5\sin\left(\omega t+\frac{\theta}{2}\right)+3\sin\left(\omega t+\frac{\theta}{3}\right)$$

$$\mathbf{A}=\frac{\mathbf{10}}{\sqrt{\mathbf{2}}}\angle\boldsymbol{\theta};\quad \mathbf{B}=\frac{\mathbf{5}}{\sqrt{\mathbf{2}}}\angle\frac{\boldsymbol{\theta}}{\mathbf{2}};\quad \mathbf{C}=\frac{\mathbf{3}}{\sqrt{\mathbf{2}}}\angle\frac{\boldsymbol{\theta}}{\mathbf{3}}$$

$$\mathbf{R}=\mathbf{A}+\mathbf{B}+\mathbf{C}$$





Tema 3 **Ecuaciones con complejos**

a. Si existe, determine X tal que $\operatorname{Im}\left[\frac{1}{2+jX}\right]=0$

$$\operatorname{Im}\left[\frac{1}{2+jX}\right]=\operatorname{Im}\left[\frac{1}{2+jX}*\frac{2-jX}{2-jX}\right]=\operatorname{Im}\left[\frac{2-jX}{4+X^2}\right]=\operatorname{Im}\left[\frac{2}{4+X^2}-j\frac{X}{4+X^2}\right]=\frac{-X}{4+X^2}$$

$$\frac{-X}{4+X^2}=0\quad entonces\quad \boldsymbol{X=0}$$

b. Determine las soluciones de $(z-1)^5=-(z+1)^5$

$$(z-1)^5=-(z+1)^5$$
$$\frac{(z-1)^5}{(z+1)^5}=-1$$
$$\left(\frac{z-1}{z+1}\right)^5=1e^{j\pi}$$

$$(v)^{5/5}=\left(e^{j(\pi+2\pi k)}\right)^{1/5}$$
$$v=e^{j\left(\frac{\pi+2\pi k}{5}\right)}$$
$$para\ \ k=\{0,1,2,3,4\}$$

Realizando una sustitución

$$v=\frac{z-1}{z+1}$$
$$v(z+1)=z-1$$
$$vz-z=-v-1$$
$$z(v-1)=-(v+1)$$
$$z=\frac{-(v+1)}{(v-1)}$$
$$z=\frac{1+v}{1-v}$$

$k=0$	$k=1$	$k=2$	$k=3$	$k=4$
$v_1 = e^{j\frac{\pi}{5}}$	$v_2 = e^{j\frac{3\pi}{5}}$	$v_3 = e^{j\pi}$	$v_4 = e^{j\frac{7\pi}{5}}$	$v_5 = e^{j\frac{9\pi}{5}}$
$z_1 = \frac{1+e^{j\frac{\pi}{5}}}{1-e^{j\frac{\pi}{5}}}$	$z_2 = \frac{1+e^{j\frac{3\pi}{5}}}{1-e^{j\frac{3\pi}{5}}}$	$z_3 = \frac{1+e^{j\pi}}{1-e^{j\pi}}$	$z_4 = \frac{1+e^{j\frac{7\pi}{5}}}{1-e^{j\frac{7\pi}{5}}}$	$z_5 = \frac{1+e^{j\frac{9\pi}{5}}}{1-e^{j\frac{9\pi}{5}}}$
$\boldsymbol{z_1 = j3.078}$	$\boldsymbol{z_2 = j0.726}$	$\boldsymbol{z_3 = 0}$	$\boldsymbol{z_4 = -j0.726}$	$\boldsymbol{z_5 = -j3.078}$

También se puede describir de la siguiente manera:

Siendo

$$\boldsymbol{g = e^{j\frac{\pi}{5}}}$$

Entonces:

$$\boldsymbol{z_{k+1} = \frac{1+g^{2k+1}}{1-g^{2k+1}} \quad para\ \ k = \{0,1,2,3,4\}}$$

Tema 4 Conversión de fasor a función trigonométrica

Dada la expresión compleja,

$$j\frac{1}{4\pi}\angle-2\theta + j\frac{1}{2\pi}\angle-\theta + \frac{1}{2} - j\frac{1}{2\pi}\angle\theta - j\frac{1}{4\pi}\angle 2\theta$$

Transfórmela a exponencial compleja, agrupe términos para expresarla como una suma trigonométrica.

$$j\frac{1}{4\pi}e^{-j2\theta} + j\frac{1}{2\pi}e^{-j\theta} + \frac{1}{2} - j\frac{1}{2\pi}e^{j\theta} - j\frac{1}{4\pi}e^{j2\theta}$$

$$\frac{1}{2} - j\frac{1}{2\pi}\left(e^{j\theta} - e^{-j\theta}\right) - j\frac{1}{4\pi}\left(e^{j2\theta} - e^{-j2\theta}\right)$$

$$\frac{1}{2} - \frac{j^2}{\pi}\left(\frac{e^{j\theta} - e^{-j\theta}}{2j}\right) - \frac{j^2}{2\pi}\left(\frac{e^{j2\theta} - e^{-j2\theta}}{2j}\right)$$

$$\frac{1}{2} - \frac{(-1)}{\pi}\sin\theta - \frac{(-1)}{2\pi}\sin 2\theta$$

$$\boldsymbol{\frac{1}{2} + \frac{1}{\pi}\sin\theta + \frac{1}{2\pi}\sin 2\theta}$$

Universidad De San Carlos Departamento de Matemática
Facultad De Ingeniería 31/08/11 Matemática Aplicada 5N

Primer Examen Parcial

Temario **B**

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Tema 1 Operaciones con complejos

$$z_1 = \frac{2i^{28} + i^{19}}{3i - 1}, \quad z_2 = \frac{1}{2} + \frac{\sqrt{3}}{2} i, \quad z_3 = 1 - 2i$$

a. $\mathrm{Re}[z_1]$

$$z_1 = \frac{2i^{28} + i^{19}}{3i - 1} = \frac{2(i^2)^{14} + i(i^2)^9}{3i - 1} = \frac{2(-1)^{14} + i(-1)^9}{3i - 1} = \frac{2 - i}{3i - 1}$$

$$z_1 = \frac{2 - i}{3i - 1} * \frac{-3i - 1}{-3i - 1} = \frac{-6i - 2 + 3i^2 + i}{9 + 1} = \frac{-2 - 3 - 5i}{10} = \frac{-5 - 5i}{10} = -\frac{1}{2} - \frac{1}{2} i$$

$$\boldsymbol{Re[z_1] = -\frac{1}{2}}$$

b. $iz_2{}^2 - z_3$

$$iz_2{}^2 - z_3 = i\left(\frac{1}{2} + \frac{\sqrt{3}}{2} i\right)^2 - (1 - 2i) = i\left(1e^{i\frac{\pi}{3}}\right)^2 - (1 - 2i) = \left(e^{i\frac{\pi}{2}}\right)\left(e^{i\frac{2\pi}{3}}\right) - (1 - 2i)$$

$$iz_2{}^2 - z_3 = \left(e^{i\frac{\pi}{2} + i\frac{2\pi}{3}}\right) - (1 - 2i) = \left(e^{i\frac{7\pi}{6}}\right) - (1 - 2i) = \left(-\frac{\sqrt{3}}{2} - \frac{1}{2} i\right) - (1 - 2i)$$

$$iz_2{}^2 - z_3 = -\frac{\sqrt{3}}{2} - \frac{1}{2} i - 1 + 2i = \frac{-\sqrt{3} - 2}{2} + \frac{3}{2} i$$

$$\boldsymbol{iz_2{}^2 - z_3 = \frac{-\sqrt{3} - 2}{2} + \frac{3}{2} i}$$

c. Valor principal en cartesianas $(1 - j)^5 (1 + j)^{1/2}$

$$(1 - j)^5 (1 + j)^{1/2} = \left(\sqrt{2} e^{-j\frac{\pi}{4}}\right)^5 (\sqrt{2} e^{j\frac{\pi}{4}})^{1/2} = \left(\sqrt{2}\right)^5 e^{-j\frac{5\pi}{4}} (\sqrt{2})^{1/2} e^{j\left(\frac{\frac{\pi}{4} + 2\pi k}{2}\right)}$$

$$para\ k = \{0, 1\}, \qquad tomando\ k = 0$$

$$(1 - j)^5 (1 + j)^{1/2} = \left(\sqrt{2}\right)^{11/2} e^{-j\frac{5\pi}{4}} e^{j\frac{\pi}{8}} = \left(\sqrt{2}\right)^{11/2} e^{j\left(-\frac{5\pi}{4} + \frac{\pi}{8}\right)} = \left(\sqrt{2}\right)^{11/2} e^{-j\frac{9\pi}{8}}$$

$$(1 - j)^5 (1 + j)^{1/2} = \left(\sqrt{2}\right)^{11/2} \left[\cos\left(\frac{-9\pi}{8}\right) + j \sin\left(\frac{-9\pi}{8}\right)\right]$$

$$\boldsymbol{(1 + j)^5 (1 - j)^{1/2} = \left(\sqrt{2}\right)^{11/2} \cos\left(\frac{9\pi}{8}\right) - j\left(\sqrt{2}\right)^{11/2} \sin\left(\frac{9\pi}{8}\right)}$$

$$\boldsymbol{(1 + j)^5 (1 - j)^{1/2} = -6.215 + j2.574}$$

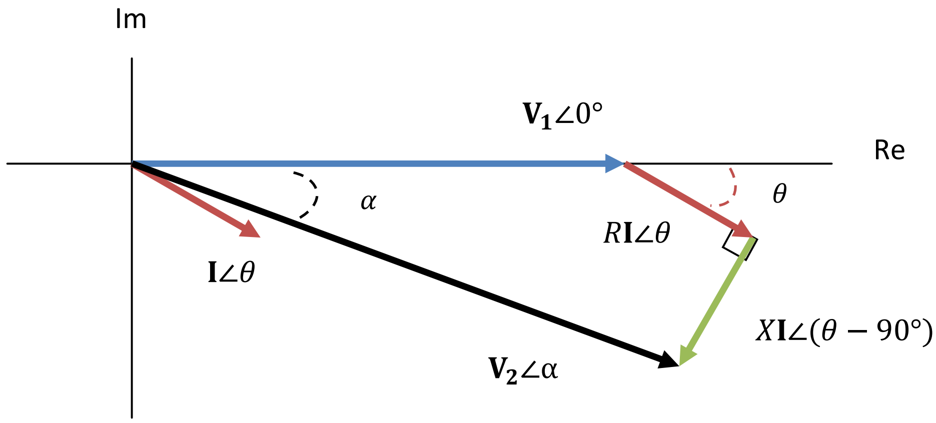
Tema 2 Lugar geométrico y fasores

a. Dibuje el diagrama fasorial de la ecuación compleja,

$$\mathbf{V_2} = \mathbf{V_1}\angle 0° + (R - jX)\mathbf{I}\angle\theta \quad suponga\ \theta < 0$$

$$\mathbf{V_2} = \mathbf{V_1}\angle 0° + R\mathbf{I}\angle\theta - jX\mathbf{I}\angle\theta$$

$$\mathbf{V_2} = \mathbf{V_1}\angle 0° + R\mathbf{I}\angle\theta + X\mathbf{I}\angle(\theta - 90°)$$



b. Mostrando los vectores que la generan, dibuje el lugar geométrico de

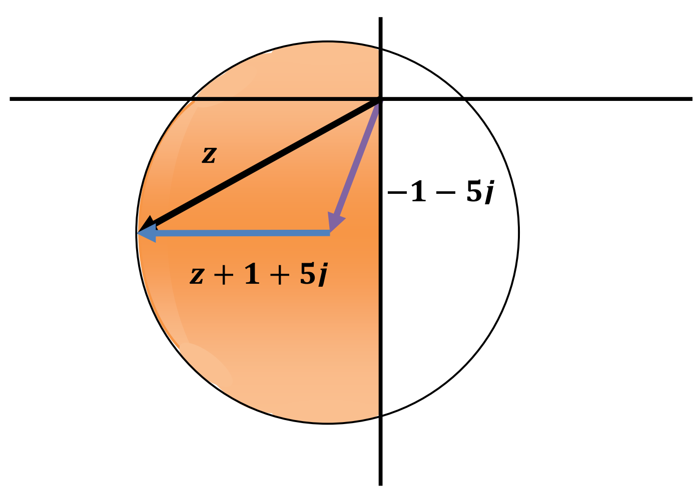
$$|z + 1 + 5j| \leq 8 \quad si\ \mathrm{Re}[z] \leq 0$$

$$z = x + jy$$

$$|x + jy + 1 + 5j| \leq 8$$
$$|(x + 1) + j(y + 5)| \leq 8$$
$$|(x + 1) + j(y + 5)|^2 \leq 8^2$$

$$(x + 1)^2 + (y + 5)^2 \leq 8^2$$
Circulo de radio 8
con centro en $(-1, -5)$

$$\boldsymbol{|z + 1 + 5j| \leq 8 \quad si\ \mathrm{Re}[z] \leq 0}$$
El lugar geométrico es la region sombreada dentro del círculo.

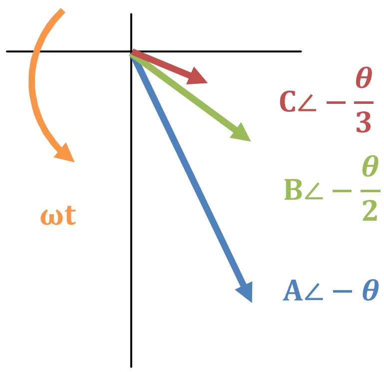


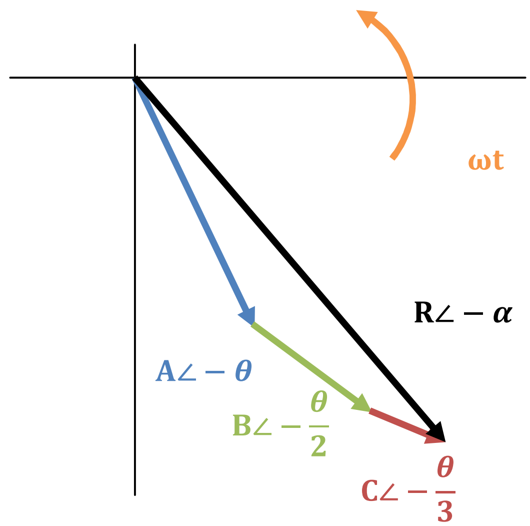
c. Determine los fasores y dibuje la suma fasorial asociada a la suma trigonométrica,

$$20\sin(\omega t-\theta)+10\sin\left(\omega t-\frac{\theta}{2}\right)+4\sin\left(\omega t-\frac{\theta}{3}\right)$$

$$\mathbf{A}=\frac{\mathbf{20}}{\sqrt{\mathbf{2}}}\angle-\boldsymbol{\theta}\quad \mathbf{B}=\frac{\mathbf{10}}{\sqrt{\mathbf{2}}}\angle-\frac{\boldsymbol{\theta}}{\mathbf{2}}\quad \mathbf{C}=\frac{\mathbf{4}}{\sqrt{\mathbf{2}}}\angle-\frac{\boldsymbol{\theta}}{\mathbf{3}}$$

$$\mathbf{R}=\mathbf{A}+\mathbf{B}+\mathbf{C}$$





Tema 3 Ecuaciones con complejos

a. Si existe, determine X tal que $\operatorname{Im}\left[\frac{1}{2-jX}\right]=0$

$$\operatorname{Im}\left[\frac{1}{2-jX}\right]=\operatorname{Im}\left[\frac{1}{2-jX}*\frac{2+jX}{2+jX}\right]=\operatorname{Im}\left[\frac{2+jX}{4+X^2}\right]=\operatorname{Im}\left[\frac{2}{4+X^2}+j\frac{X}{4+X^2}\right]=\frac{X}{4+X^2}$$

$$\frac{X}{4+X^2}=0\quad entonces\quad \boldsymbol{X=0}$$

b. Determine las soluciones de $(z-1)^5=-(z+1)^5$

$$(z-1)^5=-(z+1)^5$$
$$\frac{(z-1)^5}{(z+1)^5}=-1$$
$$\left(\frac{z-1}{z+1}\right)^5=1e^{j\pi}$$

$$(v)^{5/5}=\left(e^{j(\pi+2\pi k)}\right)^{1/5}$$
$$v=e^{j\left(\frac{\pi+2\pi k}{5}\right)}$$
$$para\ \ k=\{0,1,2,3,4\}$$

Realizando una sustitución

$$v=\frac{z-1}{z+1}$$
$$v(z+1)=z-1$$
$$vz-z=-v-1$$
$$z(v-1)=-(v+1)$$
$$z=\frac{-(v+1)}{(v-1)}$$
$$z=\frac{1+v}{1-v}$$

$k=0$	$k=1$	$k=2$	$k=3$	$k=4$
$v_1 = e^{j\frac{\pi}{5}}$	$v_2 = e^{j\frac{3\pi}{5}}$	$v_3 = e^{j\pi}$	$v_4 = e^{j\frac{7\pi}{5}}$	$v_5 = e^{j\frac{9\pi}{5}}$
$z_1 = \frac{1+e^{j\frac{\pi}{5}}}{1-e^{j\frac{\pi}{5}}}$	$z_2 = \frac{1+e^{j\frac{3\pi}{5}}}{1-e^{j\frac{3\pi}{5}}}$	$z_3 = \frac{1+e^{j\pi}}{1-e^{j\pi}}$	$z_4 = \frac{1+e^{j\frac{7\pi}{5}}}{1-e^{j\frac{7\pi}{5}}}$	$z_5 = \frac{1+e^{j\frac{9\pi}{5}}}{1-e^{j\frac{9\pi}{5}}}$
$\boldsymbol{z_1 = j3.078}$	$\boldsymbol{z_2 = j0.726}$	$\boldsymbol{z_3 = 0}$	$\boldsymbol{z_4 = -j0.726}$	$\boldsymbol{z_5 = -j3.078}$

También se puede describir de la siguiente manera:

Siendo

$$\boldsymbol{g = e^{j\frac{\pi}{5}}}$$

Entonces:

$$\boldsymbol{z_{k+1} = \frac{1+g^{2k+1}}{1-g^{2k+1}} \quad para\ \ k = \{0,1,2,3,4\}}$$

Tema 4 Conversión de fasor a función trigonométrica

Dada la expresión compleja,

$$j\frac{1}{3\pi}\angle -4\theta + j\frac{1}{\pi}\angle -\theta + \frac{1}{2} - j\frac{1}{\pi}\angle\theta - j\frac{1}{3\pi}\angle 4\theta$$

Transfórmela a exponencial compleja, agrupe términos para expresarla como una suma trigonométrica.

$$j\frac{1}{3\pi}e^{-j4\theta} + j\frac{1}{\pi}e^{-j\theta} + \frac{1}{2} - j\frac{1}{\pi}e^{j\theta} - j\frac{1}{3\pi}e^{j4\theta}$$

$$\frac{1}{2} - j\frac{1}{\pi}\left(e^{j\theta} - e^{-j\theta}\right) - j\frac{1}{3\pi}\left(e^{j4\theta} - e^{-j4\theta}\right)$$

$$\frac{1}{2} - \frac{2j^2}{\pi}\left(\frac{e^{j\theta} - e^{-j\theta}}{2j}\right) - \frac{2j^2}{3\pi}\left(\frac{e^{j4\theta} - e^{-j4\theta}}{2j}\right)$$

$$\frac{1}{2} - \frac{(-2)}{\pi}\sin\theta - \frac{(-2)}{3\pi}\sin 4\theta$$

$$\boldsymbol{\frac{1}{2} + \frac{2}{\pi}\sin\theta + \frac{2}{3\pi}\sin 4\theta}$$